MECHANISM OF HEAT AND MASS TRANSFER DURING SUBLIMATION IN A FORCED FLOW OF RAREFIED GAS

P. A. Novikov and E. A. Vagner

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The interaction of a flow of rarefied gas with the flow of vapor close to a surface undergoing sublimation is studied by a visualization method. The rate of sublimation and the intensity of convective heat transfer are expressed as functions of the velocity of the incident flow of rarefied gas.

In the design of new devices based on the principle of mass-transfer cooling for technological processes taking place in vacuo (drying, degassing, etc.) the problem of heat and mass transfer during the evaporation (sublimation) of solids in a rarefied gas flow takes on a considerable significance.

Certain problems of mass transfer cooling were treated in [1, 2], and experimental data relating to heat and mass transfer in a rarefied gas during the motion of naphthalene, water-impregnated porous ceramic, and gelatin samples were presented in [3, 4].

The absence of any qualitative picture of the mechanism governing the interaction between a flow of rarefied gas and the solid around which it flows has hitherto prevented the elucidation of certain characteristics of sublimation in the transitional pressure range (700-100 N/m^2).

Earlier investigations [5] into the interaction between solids of various shapes and an incident gas flow at normal pressures showed that the mechanism of heat and mass transfer during evaporation was mainly determined by the hydrodynamic conditions around the flow surface. It was found that the influence of the characteristic dimensions of the solid on the heat and mass transfer depended on the Reynolds number Re; for large Re numbers the whole heat and mass-transfer surface took part in the convective exchange, while for small Re numbers only part of the surface around the enveloping curve did so. With falling pressure the influence of the shape of the solid on the heat- and mass-transfer process diminished [6].

In the present investigation a flow of rarefied gas was created in the vacuum chamber by means of a conical nozzle ($2\alpha = 40°$) with a cylindrical fitting 100 mm in diameter. Air was passed into the nozzle through a system enabling the rate of flow to be regulated. The rate of flow was determined from the value recorded by a gas contour (GSB-400) and was taken as constant over the whole cross section of the fitting.

A spherical ice sample 50 mm in diameter was suspended on fine Capron filaments from a VLTK-500 balance and arranged in a gas flow on the axis of a cylinder. The temperature of the evaporation surface, the incident gas flow, and the walls of the vacuum chamber was monitored by means of copper–Constantan thermocouples. The experiments were conducted in the pressure range 260-1600 N/m^2. The total pressure in the flow was taken as equal to the static pressure, since the velocity head of the incident flow ($w_\infty = 4$ m/sec) was only 0.1% of the pressure in the chamber.

Figure 1 illustrates the $j_m = f(P, w_\infty)$ relationship, from which we see that j_m varies with P in a complicated manner, the curves agreeing qualitatively with the results of [3]. Analysis of individual terms of the material-energy balance set up for the sublimation of ice showed that the form of the $j_m = f(P, w_\infty)$ curves was completely determined by the convective component of thermal flux.

Analysis of the results of the experiments (Fig. 2) expressed in critical form showed that the relationship between the intensity of heat transfer and the rate of flow obeyed the following law

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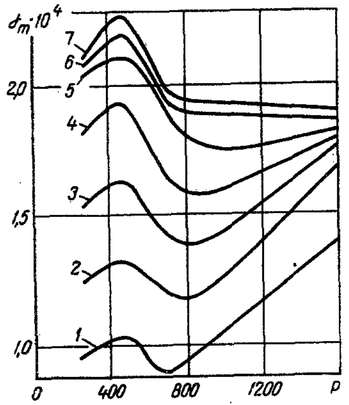

Fig. 1

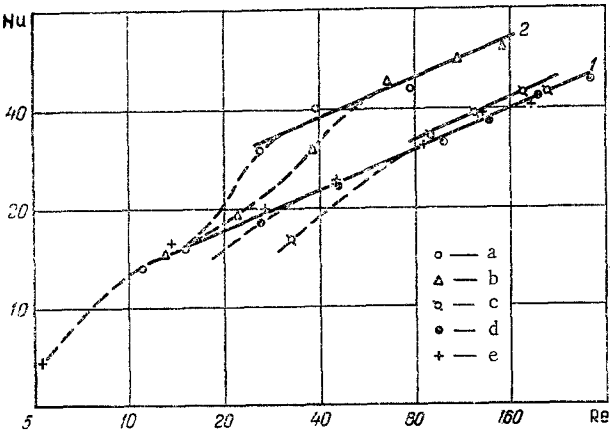

Fig. 2

Fig. 1. Intensity of evaporation j_m (kg/ $m^2 \cdot$ sec) as a function of pressure P (N/ m^2) for a flow velocity: 1) $w_\infty = 0.5$; 2) 1.0; 3) 1.5; 4) 2.0; 5) 2.5; 6) 3.0; 7) 3.5 m/ sec.

Fig. 2. Dependence of the Nu number on the Re number for the following pressures P (N/ m^2): a) P = 260; b) 460; c) 730; d) 1060; e) 1600 N/ m^2.

$$\mathrm{Nu} = A\,\mathrm{Re}^{0.4}\,. \tag{1}$$

As characteristic dimension we took the length of the body subjected to the flow πR_0. The coefficient A depends on the deformation of the vapor sheath and in the pressure range studied (P = 1600-260 N/ m^2) varies from 5.3 (curve 1) to 8.4 (curve 2).

We were able to explain the complicated dependence of the intensity of sublimation and convective heat transfer on pressure in the transitional region (Figs. 1 and 2) by analyzing visual observations recorded on motion-picture film. Close to the surface of sublimation there is a vapor sheath, the thickness of which increases as the density of the medium falls. When a current of air flows around the subliming solid, the vapor sheath is distorted, and the convective heat and mass transfer is intensified. The thickness of the vapor sheath then varies along the surface of sublimation.

Depending on the external conditions and the relationship between the velocities and densities of the air and vapor flows, the vapor sheath may undergo different degrees of distortion. For example, if the velocity of the external air flow remains constant and the pressure P falls below 460 N/ m^2, the increase in the radial flow velocity of the vapor will produce a corresponding increment in the thickness of the vapor sheath, and this will affect the intensity of the heat and mass transfer.

The broken lines in Fig. 2 indicate the ranges of heat and mass transfer not obeying this law. It appears that this deviation from the general law in the transitional pressure range may be due to a change in the shape of the vapor sheath and also to an increase in the velocity of diffusive mixing.

When a spherical solid sublimes in a rarefied space it may be regarded as a source emitting a radial flow of vapor. In the case under consideration this source is of the potential type.

When a rarefield flow of air passes around the subliming solid (this flow may also be taken as of the potential type for low velocities), the potential flows are superposed upon one another. We then obtain a new potential flow, the velocity of which at each point is equal to the sum of the velocities of the radial and external flows at the same points. In this way, by combining the two elementary flows, we obtain a new composite flow.

Figure 3 shows the contraction of the vapor sheath in the frontal part of the sphere and its expansion at the tail. The thickness of the visible vapor layer in the front of the sphere δ and its width h in the tail satisfy the relations characteristic of a three-dimensional potential flow

$$(R_0 + \delta) = 0.5\sqrt{G/\pi w_\infty} = h/4. \tag{2}$$

From this relation we may determine the position and rate of flow of the point source with which we may replace the subliming solid situated in the external rarefied flow of air. If the subliming solid has an irregular geometrical shape, it may still be approximately replaced by a source of this type.

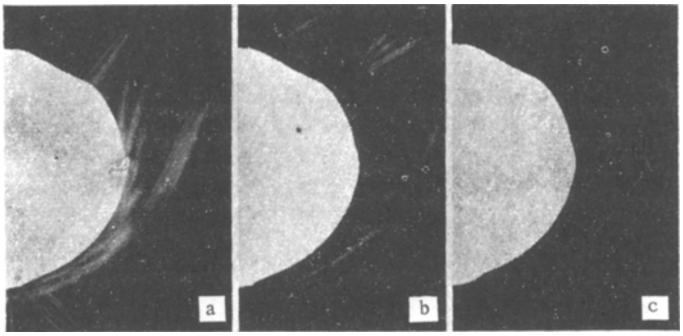

Fig. 3. Dependence of the change in the thickness of the vapor sheath around an ice sphere on the pressure P (N/m^2): a) P = 830; b) 760; c) 690 N/m^2, for w_∞ = 0.1-0.2 m/sec.

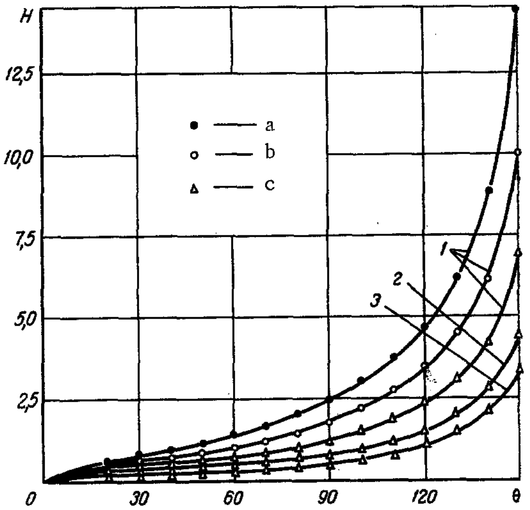

Fig. 4. Distances H (mm) by which the points in the various trajectories lie above the "zero" trajectory in relation to the parameters P (N/m^2) and w_∞ (m/sec): a) P = 260; b) 460; c) 730 N/m^2; 1) w_∞ = 0.5; 2) 1.0; 3) 1.5 m/sec.

It may be shown that a subliming solid of any geometrical shape with a potential flow passing around it will have a vapor sheath of elliptical form, in the limit approaching a sphere. In this case the picture of the hydrodynamic flow of the vapor sheath associated with a solid of arbitrary shape will differ comparatively little from that corresponding to the vapor sheath of a sphere. The effect of the distortion of the vapor sheath on the increase in evaporation rate will remain the same in the transitional pressure range for solids of other shapes also.

Let us make use of the method of conformal transformations, using the Zhukovskii imaging function [7]

$$\zeta = \frac{1}{2}\left(z + \frac{R_0^2}{z}\right) \tag{3}$$

transforming a circle into a plate. Since for the ζ-plane $\zeta = \xi + i\eta$, while for the z-plane in trigonometrical form $z = Re^{i\theta}$, we find on substituting the expression for ζ and z into (3) and separating the real and imaginary parts:

$$\xi = \frac{1}{2}\left(R + \frac{R_0^2}{R}\right)\cos\theta, \quad \eta = \frac{1}{2}\left(R - \frac{R_0^2}{R}\right)\sin\theta. \tag{4}$$

The expression for ξ and η is none other than the parametric equation of an ellipse for $R > R_0$, where R is the radius of the circle of constant concentration around the sphere constituting the mass source.

Since the majority of three-dimensional bodies occupy an intermediate position between a plate and a sphere, the vapor sheath around these bodies will acquire a spherical shape more rapidly than in the case of a plate. Calculations show that, for a permissible relative difference of 5% in the semiaxes of the ellipse, the subliming solid may be taken as a point source for distances of $L \geq 6.25\, R_0$ ($2R_0$ is the maximum linear dimension of the arbitrarily-shaped body).

For potential flows the change in the thickness of the vapor sheath around the sphere may be expressed as a function of the velocity of the external flow in analytical form. In order to analyze the two-dimensional problem of flow around a spherical source, we use the method of the superposition of potential flows.

The potential of the rate of flow passing around a sphere may be written as follows in polar coordinates

$$\varphi_1 = -w_\infty R\left[1 + \frac{1}{2}\left(\frac{R_0}{R}\right)^3\right]\cos\theta, \tag{5}$$

and the potential of the source velocity

$$\varphi_2 = -\frac{j_m}{\rho} \cdot \frac{R_0^2}{R}. \quad (6)$$

Summing the potentials and determining the projections of the velocity in polar coordinates we obtain

$$w_R = \frac{\partial \varphi}{\partial R} = -w_\infty \left[1 - \frac{1}{2}\left(\frac{R_0}{R}\right)^3\right] \cos\theta + \frac{j_m}{\rho} \cdot \frac{R_0^2}{R^2}, \quad (7)$$

$$w_S = \frac{1}{R} \cdot \frac{\partial \varphi}{\partial \theta} = w_\infty \left[1 + \frac{1}{2}\left(\frac{R_0}{R}\right)^3\right] \sin\theta. \quad (8)$$

We see by considering Eqs. (7) and (8) that, in the absence of evaporation ($j_m R_0^2/\rho R^2 = 0$), the particle trajectory passing through a specified point with constant coordinates does not depend on the velocity w_∞ and the pressure P (this is the "zero" trajectory). In the case of evaporation, the radial flow will affect the change in trajectory the more severely, the lower the velocity of the external flow.

Figure 4 shows the increment in the ordinates of the trajectories obtained in the presence of a radial flow over the corresponding points of the "zero" trajectory. We see from the figure that, with increasing velocity of the external flow, the influence of $j_m R_0^2/\rho R^2$ on the change in the particle trajectory becomes insignificant even for a pressure of P = 730 N/m^2, since the trajectory then approaches the "zero" one.

NOTATION

R_0 is the radius of the sphere;
R is the radius-vector of the trajectory; m
θ is the polar angle, deg;
w_∞ is the velocity of the external flow, m/sec;
j_m is the rate of sublimation, kg/m^2 · sec;
G is the rate of vapor flow, m^3/sec;
ρ is the density, kg/m^2;
Re is the Reynolds number;
Nu is the Nusselt number.

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